4.1 Pg. 158 #39, 41, 43, 47, 49, 55, 63

39. $y = \cos(6x+2)$

a. $y = \cos u$ and $u = 6x+2$

$y = \cos(6x+2)$ $\quad \frac{dy}{dx} = -\sin(6x+2)(6)$

$= -6\sin(6x+2)$

b. $y = \cos(2u)$ and $u = 3x+1$

$y = \cos(2(3x+1)) = \cos(6x+2)$

$\frac{dy}{dx} = -\sin(6x+2)(6) = -6\sin(6x+2)$

41. $x = 2\cos t \quad y = 2\sin t \quad t = \frac{\pi}{4}$

$x\left(\frac{\pi}{4}\right) = 2\cos\left(\frac{\pi}{4}\right) = 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2}$

$y\left(\frac{\pi}{4}\right) = 2\sin\left(\frac{\pi}{4}\right) = 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2}$ $\quad (x, y) = (\sqrt{2}, \sqrt{2})$

$\frac{dx}{dt} = -2\sin t$ at $t = \frac{\pi}{4}$ $\quad \frac{dx}{dt} = -2\sin\left(\frac{\pi}{4}\right) = -2\cdot\frac{\sqrt{2}}{2} = -\sqrt{2}$

$\frac{dy}{dt} = 2\cos t$ at $t = \frac{\pi}{4}$ $\quad \frac{dy}{dt} = 2\cos\left(\frac{\pi}{4}\right) = 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2}$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{dy}{dx} = \frac{\sqrt{2}}{-\sqrt{2}} = -1$$

Tangent line equation $\quad y - \sqrt{2} = -(x - \sqrt{2})$

43. $x = \sec^2 t - 1 \quad y = \tan t \quad t = \frac{-\pi}{4}$

$x\left(\frac{-\pi}{4}\right) = \left(\sec\left(\frac{-\pi}{4}\right)\right)^2 - 1 = \left(\sqrt{2}\right)^2 - 1 = 2 - 1 = 1$

$y\left(\frac{-\pi}{4}\right) = \tan\left(\frac{-\pi}{4}\right) = -1$

$\frac{dx}{dt} = 2\sec t \sec t \tan t \quad \frac{dx}{dt} = 2\left(\sec\left(\frac{-\pi}{4}\right)\right)^2 \tan\left(\frac{-\pi}{4}\right)$

$= 2\left(\sqrt{2}\right)^2(-1) = -4$

$\frac{dy}{dt} = \sec^2 t \quad \frac{dy}{dt} = \left(\sec\left(\frac{-\pi}{4}\right)\right)^2 = \left(\sqrt{2}\right)^2 = 2$

tangent line equation

$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{2}{-4} = \frac{-1}{2}$

$y - -1 = \frac{-1}{2}(x-1)$

$y + 1 = \frac{1}{2}(x-1)$

47. $x = t - \sin t \quad y = 1 - \cos t \quad t = \frac{\pi}{3}$

$x\left(\frac{\pi}{3}\right) = \frac{\pi}{3} - \sin\left(\frac{\pi}{3}\right) = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$

$y\left(\frac{\pi}{3}\right) = 1 - \cos\left(\frac{\pi}{3}\right) = 1 - \frac{1}{2} = \frac{1}{2}$

$\frac{dx}{dt} = 1 - \cos t \quad \frac{dx}{dt} = 1 - \cos\left(\frac{\pi}{3}\right) = 1 - \frac{1}{2} = \frac{1}{2}$

$\frac{dy}{dt} = \sin t \quad \frac{dy}{dt} = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{dy}{dx} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \sqrt{3}$

$y - \frac{1}{2} = \sqrt{3}\left(x - \frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$

49. $y = \sin t \qquad x = t^2 + t$

a. $\frac{dy}{dt} = \cos t \qquad \frac{dx}{dt} = 2t+1 \qquad \frac{dy}{dx} = \frac{\cos t}{2t+1} = y'$

b. $\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{(2t+1)(-\sin t) - (\cos t)(2)}{(2t+1)^2}$

$= -\frac{(2t+1)(\sin t) + 2\cos t}{(2t+1)^2}$

c. $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(y') = \frac{dy'}{dx} = \frac{\left(\frac{dy'}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\left(\frac{(2t+1)(-\sin t) - (\cos t)(2)}{(2t+1)^2}\right)}{2t+1}$

$= -\frac{(2t+1)\sin t + 2\cos t}{(2t+1)^3}$

55. $y = 2\tan\left(\frac{\pi}{4}x\right) \qquad x = 1 \qquad y(1) = 2\tan\left(\frac{\pi}{4}\right) = 2(1) = 2$

$\frac{dy}{dx} = 2\sec^2\left(\frac{\pi}{4}x\right) \cdot \frac{\pi}{4} = \frac{\pi}{2}\sec^2\left(\frac{\pi}{4}x\right)$

at $x = 1 \quad \frac{dy}{dx} = \frac{\pi}{2}\left(\sec\left(\frac{\pi}{4}\right)\right)^2 = \frac{\pi}{2}\left(\sqrt{2}\right)^2 = \frac{\pi}{2} \cdot 2 = \pi$

Tangent line equation $y - 2 = \pi(x-1)$

Normal line equation $y - 2 = -\frac{1}{\pi}(x-1)$

63. $s = \sqrt{1+4t}$

$$v(t) = \frac{1}{2}(1+4t)^{-\frac{1}{2}}(4) = \frac{2}{\sqrt{1+4t}}$$

$$a(t) = 2\cdot\frac{1}{2}(1+4t)^{-3/2}(4) = \frac{4}{\sqrt{(1+4t)^3}}$$

$$v(6) = \frac{2}{\sqrt{1+4(6)}} = \frac{2}{\sqrt{25}} = \frac{2}{5} \text{ m/sec}$$

$$a(6) = \frac{4}{\sqrt{(1+4(6))^3}} = \frac{4}{\sqrt{25^3}} = \frac{4}{25\sqrt{25}}$$

$$= \frac{4}{25\cdot 5} = \frac{4}{125} \text{ m/sec}^2$$